

# Periodically-driven cold atoms: the role of the phase

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**Abstract.** Numerous theoretical and experimental studies have investigated the dynamics of cold atoms subjected to time periodic fields. Novel effects dependent on the amplitude and frequency of the driving field, such as Coherent Destruction of Tunneling have been identified and observed. However, in the last year or so, three distinct types of experiments have demonstrated for the first time, interesting behaviour associated with the *driving phase*: i.e. for systems experiencing a driving field of general form  $V(x)\sin(\omega t + \phi)$ , different types of large scale oscillations and directed motion were observed. We investigate and explain the phenomenon of Super-Bloch Oscillations (SBOs) in relation to the other experiments and address the role of initial phase in general. We analyse and compare the role of  $\phi$  in systems with homogeneous forces ( $V'(x) = \text{const}$ ), such as cold atoms in shaken or amplitude-modulated optical lattices, as well as non-homogeneous forces ( $V'(x) \neq \text{const}$ ), such as the sloshing of atoms in driven traps, and clarify the physical origin of the different  $\phi$ -dependent effects.

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## 1. Introduction

Experiments on cold atoms in optical lattices subject to time-periodic perturbations [1, 2, 3, 4, 5] have provided exceptionally clean demonstrations of coherent phenomena predicted in theoretical studies [6, 7, 8, 9, 10, 11, 12, 13, 14]. Of especial note are the phenomena termed Coherent Destruction of Tunnelling (CDT) [7, 8, 9] or Dynamic Localization (DL) [6].

In particular, if an oscillating potential  $V(x, t) = Fx \cos \omega t$  is applied, it was found that the tunnelling amplitudes  $J$  of the driven atoms take an effective, renormalized, value:

$$J_{\text{eff}} \propto J \mathcal{J}_0 \left( \frac{Fd}{\hbar \omega} \right), \quad (1)$$

where  $d$  is the lattice constant and  $\mathcal{J}_0$  denotes a zero-th order ordinary Bessel function. In the CDT ( $\omega \gg J$ ) regime, there is a complete cessation of tunnelling at the zeros of the Bessel function; in the DL regime, the atoms make a periodic excursion but return to their original position if  $\mathcal{J}_0 \sim 0$  [15].

The above showed that even the one-particle dynamics can exhibit non-trivial effects arising from matter-wave coherence. The potentials were realized with ultracold atoms in shaken optical lattices; Eq.(1) was demonstrated and investigated experimentally in a series of ground-breaking experiments [1, 3]. The effects of interactions have also been investigated [5, 16, 17, 18, 19] such as AC-control of the Mott-Insulator to Superfluid phase transition, including even triangular optical lattices [20].

The experiments above identified effects which depend on the frequency and amplitude of the driving field, not its initial phase. However, in the last year or so, three distinct experiments on periodically driven cold atoms have identified effects due to the phase; in other words, for driving of form  $V(x, t) \propto \sin(\omega t + \phi)$ , the dynamics was found to depend strongly on  $\phi$ . For example, for atoms in shaken lattices, [21, 22] were able to realize both directed motion as well as large oscillations, occurring over hundreds of sites, which were termed “Super-Bloch Oscillations” (SBOs) [22]. These SBOs were analyzed in [23], including also the effects of weak interactions (mean-field regime). The general assumption in theoretical analyses has been that the external field-dependence of the wavepacket mean group velocity in these cases is entirely contained in the Bessel function argument, with no dependence on  $\phi$ . However, the experiment indicates that there is a strong dependence on this phase.

In a second example, [24], the tunneling amplitude of the lattice was modulated. The result was a global motion (non-zero velocity of the atomic wavepacket), strongly dependent on  $\phi$ . The third example did not involve optical lattices or tunneling but looked at a BEC “sloshing” in a time-averaged orbiting potential (TOP) trap. Experiments showed [25] using a 2-dimensional set up, that the amplitude of oscillations of atoms in traps has a strong dependence on phase. Related theoretical work on the atoms oscillating in traps was carried out by [26, 27].

The atoms in traps are also driven with potentials of the form (in 1-dimension)  $V(x) \sin(\omega t + \phi)$ . Yet they differ in essential ways. Because the potential is not linear, the force is not uniform; the time-averaged motion becomes strongly coupled to the phase; the trap analysis always requires application of an adiabatic separation between fast  $\omega$  and slow mean motion. In addition, they also differ from the optical lattice systems because the latter are band-Hamiltonians, with an effective kinetic energy  $-J \cos p$  (in a classical-image Hamiltonian). Such differences mean that effects due to  $\phi$  are more subtle in the lattices; thus for the latter, phase effects have been neglected for several years, while they quickly became apparent in the trap systems.

However, at present, the physical role of the initial phase in all these experiments does not seem too clear. Recent analyses consider the effect of the phase in terms of a shift in the origin of time for shaken lattices [4] or that the effects arise due to the abrupt amplitude jump which occurs when the field is initially switched-on [25] at  $t \simeq 0$ .

We have recently found [28], that in order to explain the experiments [21, 22], a phase correction  $\Phi(F) = F_\omega \cos \phi - n(\phi + \frac{\pi}{2})$  must be considered in an effective dispersion relation. This explains a field dependent shift in the phase of SBOs. It accounts also for a separate regime of directed motion. Here we analyse these SBOs in relation to other comparable experiments with cold atoms which have also observed other types of global motion and large scale oscillations such as trap oscillations or amplitude-modulated dynamics. More broadly, the aim of this work is to clarify the role of the initial phase in such systems and to consider the validity of proposed models and assumptions in these related experiments.

It is worth stressing that for shaken lattices and SBOs, the strongest effects occur when the system evolves from  $p = 0$  with a smooth pure  $\sin \omega t$  drive, where there is no amplitude jump or even a phase-jump at  $t = 0$ . Yet below we consider only oscillations where the phase is well-defined in each oscillation. Thus  $\phi$  has to be set on a timescale fast compared with  $T = 2\pi/\omega$ . Variants with a slower ramping-up procedure depend on the particular protocol and we do not consider these.

Below we consider systems with homogeneous forces ( $V'(x) = \text{const}$ ) separately from those for which the forces are position dependent.

## 2. Inhomogeneous systems: Atoms in Traps

We consider first the classical dynamics for a Hamiltonian of form:

$$H(x, t) = H_0(p) + V(x)F(t). \quad (2)$$

Here  $F(t) = F(t + T)$  is a time periodic driving term and  $H_0(p) = \frac{p^2}{2m}$  for traps or  $H_0(p) = -J \cos p$  (for a one-body image Hamiltonian for a particle in a band). One may also consider an additional time independent potential, i.e.  $H_0(p, x) = H_0(p) + V_0(x)$  but for the essential physics, this is not necessary for now.

Thus there is a classical force:

$$\dot{p} = -V'(x)F(t). \quad (3)$$

If one can neglect the time evolution of  $x(t)$  in the interval  $[0 : t]$ , we have a momentum shift on the initial momentum  $p_0$ :

$$p(t) = p_0 - V'(x_0) \int_0^t F(t) dt, \quad (4)$$

where  $x_0 = x(t = 0)$

One instance is the situation, well-studied experimentally [29, 30, 31, 32, 33, 34], of  $\delta$ -kicked cold atoms where an optical lattice potential is abruptly switched-on for a very short time and where  $F(t) = \Sigma_n \delta(t - nT)$ . For a single kick, and  $T = 1$ , the shift is  $p = p_0 - V'(x)$ . There is an instantaneous impulse applied to the atoms. It is well-known that, provided the switch on is sufficiently fast and the pulse is of short duration, the dynamics is insensitive to the pulse shape. However, the shift is position dependent; for repeated kicks, this can have drastic consequences including chaos, for which kicked atoms are a leading paradigm.

Another instance of a position-dependent driving, corresponds to atoms in traps [26, 27], where  $V(x)$  represents, for example, a harmonic trap of frequency  $\Omega$  and  $F(t) = \sin(\omega t + \phi)$  represents a sinusoidal drive. In the case  $\omega \gg \Omega$  one can consider a period-averaged mean motion for the slow oscillation in the well, with coordinate  $X(t)$ , as well as a “micromotion” characterized by the faster  $\omega$  timescale. Adiabatic separability means that one may consider these motions separately. In [26, 27] an effective shift of the mean-momentum  $P_0 = m\dot{X}(0)$  was identified. In the present notation, one would obtain an effective phase-dependent shift of the period-averaged momentum:

$$P'_0 = p_0 + \frac{V'(x_0)}{\omega} \cos \phi. \quad (5)$$

A mean-momentum shift

$$\Delta P(x_0) = \frac{V'(x_0)}{\omega} \cos \phi \quad (6)$$

is zero for  $\phi = \pi/2$  or  $\phi = 3\pi/2$ . But it is maximal for  $\phi = 0, \pi$  the case where the  $F(t) = \pm \sin \omega t$ ; for this case, the driving field starts smoothly from zero. This clearly indicates that this case represents essentially different physics from the  $\delta$ -kicked particles. The shift is unrelated to the abruptness of the change in the Hamiltonian amplitude at  $t = 0$ , but rather to a “slippage” discussed in [26, 27] between the slow mean motion and the fast “micromotion”. It makes little sense to define period-averaged motion on a timescale shorter than of order  $T = 2\pi/\omega$ : the system requires a finite time to “notice” that the mean momentum has shifted; for the  $\delta$ -kicked particle, in contrast, there is a real impulse, applied instantaneously on a timescale  $\ll T$ . A formal way of stating this is that a  $\delta(t)$  function implies a finite impulse at  $t \simeq 0$ , while the Heaviside step function switch-on of a  $\sin(\omega t + \phi)$  drive gives zero impulse at  $t \simeq 0$  and is thus not significant to the dynamics.

Nonetheless, the shift  $\Delta P(x_0)$  has real physical implications as it can reduce or increase the kinetic energy of the mean motion, depending on its sign. It leads to an increase or decrease of the amplitude of the harmonic oscillations of the particle in the

trap (averaged over one period). Ridinger and Davidson [26, 27], for instance considered these and other examples of traps (Eqs. (15) and (16) therein).

These effects were demonstrated experimentally in atoms sloshing in a TOP trap [25]. The experiments however, used a 2-dimensional set-up. Even for  $\phi = 0$ , a rotating field  $(B \cos \omega t, B \sin \omega t)$  is applied in the  $(x, y)$  directions. There is never a situation where a pure sine drive is applied and there is always an initial “jump” in field amplitude in at least one coordinate. Thus a convenient view (Ref. [25]) is to reset the large scale oscillation (macromotion) amplitude coordinates at the end of the (near-instantaneous) switch-on of the field. The macromotion is thus attributed to the sudden switch-on. However, one should stress that the underlying physics can emerge only on the timescale of  $T$ , rather than being due to the abrupt initial switch-on procedure.

### 3. Homogeneous systems: shaken optical lattices

The shaken optical lattices represent a different class of Hamiltonians. The corresponding classical Hamiltonian is:

$$H(x, t) = -J \cos p - Fx \sin(\omega t + \phi). \quad (7)$$

A key difference is that  $V'(x) = F$  is independent of position. While the particle does display Bloch Oscillations (SBOs) with the period  $T_B$ , there is no separation of timescales since  $T_B \simeq T$ . There is no restriction to high frequency driving. Further, there is a decoupling between the instantaneous phase of the Bloch Oscillations and the phase, since the initial position  $x(t = 0)$  is immaterial: the force is uniform. The initial wavepacket is even delocalized over several lattice sites. From the classical equations of motion,  $p = p_0 + F \int_0^t \sin(\omega t' + \phi) dt'$  and an effective momentum shift arises from the integrand at  $t = 0$ :

$$p(t) = p_0 + \frac{F}{\omega} \cos \phi - \frac{F}{\omega} \cos(\omega t + \phi). \quad (8)$$

The effective shift is, however, position independent:  $p'_0 = p_0 + \frac{F}{\omega} \cos \phi$ . It is possible to then consider the effect of the phase as simply a corresponding displacement in the origin of time as suggested in [4], but only in the momentum shifted frame of  $p'_0$ .

Once again, for the completely smooth switching on of the driving Hamiltonian (pure  $\sin \omega t$  drive) the shift is a maximum. And here it is not useful to think of an initial “jump” either in amplitude or phase. Taking the experimental value  $p_0 = 0$  and  $\phi = 0$ ,

$$\langle p(t) \rangle = \frac{F}{\omega} \langle 1 - \cos \omega t \rangle. \quad (9)$$

We see that the momentum now oscillates about a shifted non-zero value, but the process is not related to the abruptness of the switch-on, which is perfectly smooth. The shift in the average momentum becomes apparent only on the timescale of  $T$ . This is very different from the  $\delta$ -kicked particle, where a real instantaneous impulse is imparted.

It is also different from the trap, where through  $V'(x_0)$ , the position at the instant the phase is set is important.

One may solve Hamilton's classical equations of motion to obtain a period averaged group velocity:

$$v_g \simeq -J\mathcal{J}_0(F_\omega) \sin(p + F_\omega \cos \phi), \quad (10)$$

where  $F_\omega = F/\omega$ .

In the shaken lattice, there is no dependence on initial position  $x_0$ . Thus we conclude that the directed motion is not related to the sudden switch-on of the field. It is best understood as a process more akin to ratchet physics and the de-symmetrization of the drive. The time-asymmetric  $\sin \omega t$  drive generates a ratchet current (since the average force is zero, there is no net bias: thus this fulfils one of the criteria for ratchet motion, though not the stricter one of "rectification of fluctuations", so should perhaps be termed simply "directed motion"). The symmetric drive  $\cos \omega t$  produces no ratchet motion, while intermediate values of  $\phi$  produce intermediate behaviour. The final current is similar to that obtained in the linear-ramping study of Ref.[35].

The above discussion explains only directed motion and its  $\phi$  dependence. For Super-Bloch Oscillations (SBOs), one has an additional static field. One must consider in addition the quantum resonance between  $\omega$  and the gap between energy levels in adjacent wells. We thus consider a quantum treatment, within a Floquet-theory framework (full details are in [28]).

We consider the total Hamiltonian:  $H(t) = H_0 + H_F(t)$ , where

$$H_0 = -\frac{J}{2} \sum_{j,\sigma} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{H.c.}). \quad (11)$$

$H_0$  corresponds to the non-interacting limit of a variety of Hamiltonians with nearest-neighbour hopping (Hubbard, Bose-Hubbard, magnons in Heisenberg spin chains, etc.) It represents any spatially periodic potential characterized by energy eigenfunctions  $\phi_{mk}$ , with band index  $m$  and wavenumber  $k$ , thus  $H_0 \phi_{mk} = E_m(k) \phi_{mk}$ . We restrict our one-particle problem to the lowest band  $m = 1$ ; taking  $E(k) \equiv E_{m=1}(k)$ , the energy dispersion  $E(k) = -J \cos kd$ .

We take  $H_F(t) = -f(t)x$  where  $f(t) = F_0 + F \sin(\omega t + \phi)$ ; in general it comprises both a static field and a sinusoidally oscillating field with an arbitrary phase  $\phi$ . The result of the driving is a time-dependent wavenumber [4]:

$$q_k(t) = k + \frac{1}{\hbar} \int_0^t d\tau f(\tau) = k + \frac{F}{\omega} \cos \phi - \frac{F}{\omega} \cos(\omega t + \phi) \quad (12)$$

The stationary states of the system are its Floquet states, the analogues of Bloch waves in a temporally periodic system. They are given by  $\psi_k(x, t) = u_k(x, t) \exp[-\frac{i}{\hbar} \epsilon(k)t]$ , where  $u_k(x, t) = u_k(x, t + T)$ , and the period  $T = 2\pi/\omega$ . The non-periodic phase-term is characterized by the so-called "quasienergy"  $\epsilon(k)$ .

The presence of the static linear field in general destroys the band dynamics; however, here we consider the so-called resonant driving case, for which  $F_0 d = n\hbar\omega$ ,

(where the driving compensates for the energy offset between neighbouring wells in the lattice, restoring the band structure). In this case, other studies, while neglecting  $\phi$ , have found that the renormalization of tunnelling is  $n$ -dependent:

$$J_{\text{eff}} \propto J \mathcal{J}_n \left( \frac{Fd}{\hbar\omega} \right), \quad (13)$$

where  $d$  is the lattice constant and  $\mathcal{J}_n$  denotes an ordinary Bessel function and  $n = 0, 1, 2, \dots$ . Below we take  $\hbar = 1$  and  $F_\omega = Fd/\omega$ .

For the purposes of calculating to group velocities, we define an effective quasienergy dispersion:

$$\epsilon(k) = -\frac{J}{T} \int_0^T \cos(q_k(t)d) dt \quad (14)$$

by means of a period-average.

Below we also consider the case of slight detuning for which  $F_0 d = (n + \delta)\omega$ , with  $\delta \ll 1$ , associated with SBOs; however, for the slight-detuning case  $\delta \neq 0$ , we assume that the time-dependence due to the  $\delta\omega t/d$  remains negligible over one period  $T$ . Thus we take it out of the period-average entirely and, as shown in [28], Eq. (14) becomes:

$$\epsilon(k) \simeq -J \mathcal{J}_n(F_\omega) \cos \left[ kd + \delta\omega t + F_\omega \cos \phi - n\left(\phi + \frac{\pi}{2}\right) \right]. \quad (15)$$

The above represents an *effective* dispersion relation, but which *oscillates slowly in time* with a period  $T_{\text{SBO}} = 2\pi/(\delta\omega) \gg T_B$  where  $T_B \propto 1/F_0$  is the Bloch period. They correspond to the ‘‘Super-Bloch Oscillations’’ investigated by Refs. [13, 14, 23, 21, 22]. Even at resonance  $\delta\omega = 0$ , Eq. (15) differs from previous expressions by the phase-shifts  $\Phi(F) = F_\omega \cos \phi - n(\phi + \frac{\pi}{2})$ . Evaluating Eq. (15) for the  $n = 1$  case, we obtain a period-averaged group velocity:

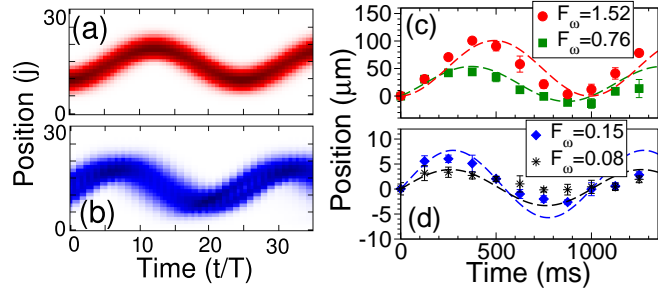
$$v_g = \frac{\partial \epsilon}{\partial k} \simeq -Jd \cos(kd + F_\omega \cos \phi - \phi + \delta\omega t) \mathcal{J}_1(F_\omega). \quad (16)$$

The above expression accounts for a range of experimental features. However, the most interesting one is that the SBOs begin with a field-dependent phase. Figure 1 demonstrates this for experimental field values  $F_\omega = 0.08, 0.15, 0.76$  and  $1.52$  (compared with one-particle Hubbard numerics). The graphs show that Eq. (16) reproduces quite well the experimental values of Ref. [22]. They show clearly the displacement of the first maximum, seen in the experiment as well as the order of magnitude variation in amplitude. Such a field-dependent shift is also apparent in the results of Ref. [21].

Finally, we consider the case of experiments with amplitude modulated lattices, which do not show renormalisation of tunneling, but can also produce directed motion [24]. The experiments also used a static linear field  $F_0 x$ . An experimental group velocity  $v_g \propto J \cos(kd - \phi)$ , for the  $n = 1$  resonance case  $F_0 d = \hbar\omega$ , was measured. Curiously, the effective momentum shift  $F_\omega \cos \phi$  seen in other driving systems is no longer present.

We find that the observed behavior in [24] is consistent with an effective classical Hamiltonian for the homogeneous force system:

$$H(x, t) = -J[1 + \alpha \sin(\omega t + \phi)] \cos p + F_0 x. \quad (17)$$



**Figure 1.** (a) and (b) One-particle solutions of the Hubbard Hamiltonian with the driving term for  $L = 30$  lattice sites, showing the field-dependence of the phase of Super-Bloch Oscillations [28]. Upper/lower panel correspond to  $F_\omega = 1.52$  and  $0.15$ ; both have  $\phi = \pi$ . For illustration purposes, the theoretical amplitudes were equalized by choosing  $J$ -values which equalize  $J_{\text{eff}} \simeq J \mathcal{J}_1(F_\omega)$ . (c) and (b) Show that good agreement is obtained between experiment and the Hubbard numerics, as well as the analytical formula [28]: integrating Eq. (16) (dashed lines) reproduces well both the amplitude and phase of SBO experimental data (symbols) of Ref. [22], using  $\delta\omega = 2\pi/1000$  ( $\equiv -1$  Hz detuning),  $\phi = \pi$  and  $Jd/(\hbar\delta\omega) = 90 \mu\text{m}$ .

Solving for Hamilton’s classical equations of motion, we see that  $p(t) = p_0 - F_0 t$  no longer has a phase dependence. For simplicity, we take  $\hbar = d = 1$  and thus  $F_0 = \omega$ . Then we easily see that the period averaged velocity  $\langle \Delta x \rangle / T = \frac{1}{T} \int_0^T \dot{x}(t) dt = -\frac{1}{2} J \alpha \cos(p_0 - \phi)$ . In other words, even with zero momentum shift, in this case a non-zero phase correction can arise.

#### 4. Conclusion

In this study, we have attempted to clarify the role of driving phase in the dynamics of homogeneous and inhomogeneous-force systems. To this end we have compared recent experimental results on homogeneous systems (shaken lattices) and inhomogeneous systems (atoms oscillating in traps).

It is tempting to draw analogies between the “slow” SBOs with the macromotion or “sloshing” seen in traps [25]. However, in the latter, the effect of the phase is manifested directly in the amplitude of the oscillations in the trap. In the shaken lattices, the manifestation of the phase is more subtle. The amplitude of SBOs is independent of  $\phi$ , only their phase is affected. Unlike the trap systems, their time-dependence does not arise from the period-average integral. Nonetheless, it seems clear that in order to account for the experiments it is necessary to include the phase-correction  $\Phi(F)$  in an effective dispersion, used to obtain group velocities.

What all these systems do have in common is that the main effect of the initial phase is to modify the global centre of mass dynamics of the wavepacket. This is in contrast to previous experiments on, for example, DL, which showed diffusive spreading of the wavepacket rather than large-scale motion, on scales of many sites.



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